DOCUMENT RESUME

ED 184 207 EA 012 514

AUTHOR Dembowski, Frederick L.
TITLE An Integer Programming Approach to School District Financial Management.
PUB DATE 80
NOTE 24p.: Paper presented at the Annual Meeting of the American Educational Research Association (Boston, MA, April 7-11, 1980).

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Computer Programs; Credit (Finance); Elementary Secondary Education; Financial Policy; Interest; *Investment; *Money Management; School Accounting; *School Districts

ABSTRACT

Because of the nature of school district cash flows, there are opportunities for investing surplus cash and the necessity to borrow cash in deficit periods. The term structure of interest rates makes the manual determination of the optimal financial package impossible. In this research, an integer programming model of this cash management process was formulated and empirically tested. The model determines the optimal timing and amounts of any investments or loans necessary, resulting in the maximization of net interest earnings. The universal use of this model could result in millions of additional dollars annually in American school district interest revenues. (Author)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED184207

U S DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

AN INTEGER PROGRAMMING APPROACH TO SCHOOL DISTRICT FINANCIAL MANAGEMENT

by

Frederick L. Dembowski*

Spring, 1980

A Paper Presented at the Annual Conference of the American Educational Research Association (AERA), Boston, Massachusetts, April, 1980.

*The author is an Assistant Professor of Educational Administration at the State University of New York at Albany.

EA 012 514

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

F. Dembowski

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

Introduction

The management of school districts is big business. It is not unusual for a school district to have an operating budget of over $1 million, and many large school districts have budgets exceeding $25 million. Corporations in the private sector with budgets of this magnitude require sizeable business staffs to plan and manage the entities' fiscal operations. Because of economic incentives, efficiency in the financial management arena is very important, not only for the private sector, but for the public sector as well. School districts cannot go out of business but there are still important reasons for promoting efficiencies in financial management and planning. Increasingly important is the need to ease the tax burden of the residents of the district. This paper focuses on the issue of school district financial management and planning. It shows how an optimization technique, integer programming, may be used to aid in the development of a sound and efficient financial package for the school district.

The Background of School District Financial Planning

There is a need for financial planning in school district management because of the nature of the cash flows. School districts, like businesses in the private sector, have revenues (cash inflows) and expenditures (cash outflows). Because of differences in the timing and amount of inflows and outflows, the school district may be faced with periods of cash surplus when revenues exceed expenditures, or cash deficit when expenditures exceed revenues.

School districts receive most of their revenues through two sources: property taxes and state aid payments. Both of these payments accrue to the district in large lump sums periodically throughout the fiscal year. The property tax receipts are usually distributed to school districts once or twice a year. Because these receipts comprise between 40% to 60% of the total revenue received by the district, these annual or semi-annual revenue payments are quite large. The school district administration may anticipate the amount of property taxes very accurately because

it determines the tax levy during the budgeting process.

State aid payments are usually the second largest source of school district revenue. The timing and amount of state aid payments vary from state to state; however, these payments are also quite large. Again, the amounts of state aid may be anticipated with a high degree of certainty. Together property tax receipts and state aid payments comprise most of the school district's receipts.

School district expenditures may also be anticipated. School district expenditures are comprised primarily of salaries and associated fringe benefit payments. The typical school district budget allots approximately 80% of total operating expenditures for these purposes. Once the salary schedule is negotiated and the number of employees is determined, these expenditures may be calculated. Except for the Summer months, the cash outflow from the school district is quite stable, with June being the highest expenditure month because of social security and tax payments to the federal government. Other major expenditures include debt service payments, utility bills, equipment, and supply costs. All of these expenses may be anticipated in the planning period. In addition, the school district has a good deal of latitude as to when it wishes to pay many bills.

The most important consideration for this discussion is that the school districts are able to anticipate both the timing and amount of their major revenues and expenditures. This enables the school district to develop a cash flow schedule (commonly known as a "cash flow chart"). The rest of this article will be concerned with a discussion of the cash flows and financial management of an example school district.

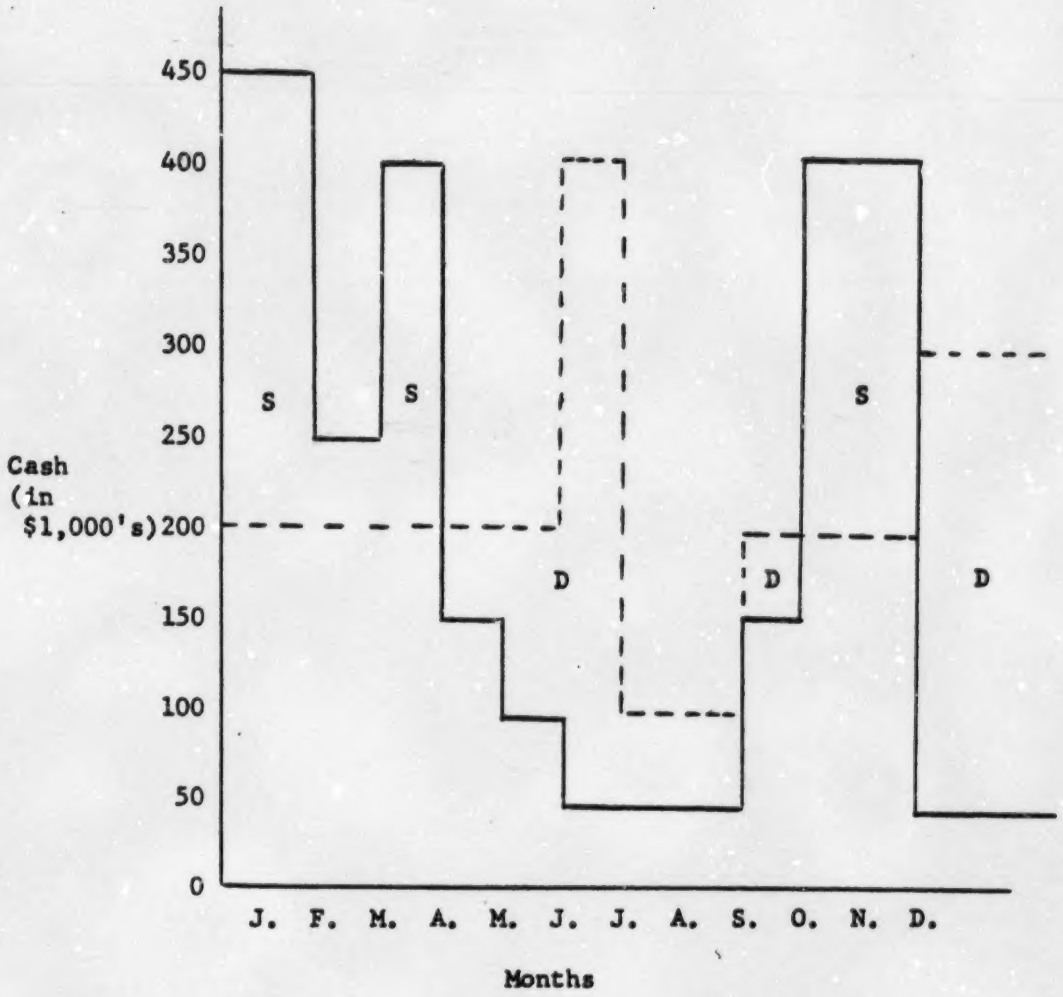
Table 1 presents the monthly anticipated inflows and outflows of the school district. October and January cash inflows (revenues) are quite large because of property tax receipts while the November and March inflows are large because of state aid payments. The cash outflows (expenditures) are stable for most months

at $200,000, with a high in June and low expenses in the Summer months. Because of the differences in the timing and amounts of the cash inflows and outflows, the school district experiences periods of cash surplus and deficits. Figure 1 presents a graphical display of these cash flows. January through March are months of cash surplus, denoted by "S". From April through October, the school district has more expenditures than revenues. These periods of cash deficit are denoted by "D".

The Problem

The primary task of the financial planners of the district is to determine when any investments can be made, and if the school district will need to borrow money. This financial management task is usually approached in the following manner. First, the net cash flow is determined by substracting the cash outflows from the cash inflows. The netflows for each month for an example school district are given on Table 1. In January, the district has a net flow of +$200,000. This cash should be invested in interest-bearing assets such as Certificates of Deposit. Since the month of April has a cash deficit of $50,000, $50,000 of the $200,000 could be invested from January to April to cover that deficit. Another $100,000 could be invested from January to May and then used to cover the $100,000 deficit in that month with the remaining $50,000 from January invested until June. Because revenues equal expenditures, this process could be followed with all surplus cash invested to mature during months of cash deficit.

This hypothetical investment process would continue until the month of July. In July, the school district needs $50,000 to meet current expenditures, but all available revenue has been expended. This deficit situation continues each month until October when the property tax revenues arrive. By that time the district will have accumulated a deficit of $150,000. Clearly, the district must borrow cash to cover these deficits.



S - indicates a cash surplus
D - indicates a cash deficit

---------- expenditures
———————— revenues

Figure 1.

The Cash Budget and Cash Flow Pattern of the Sample District

TABLE 1

The Cash Flow Pattern of the Sample District

Period	0	1	2	3	4	5	6	7	8	9	10	11	12
Month	Jan. 1	Feb. 1	Mar. 1	Apr. 1	May 1	June 1	July 1	Aug. 1	Sept. 1	Oct. 1	Nov. 1	Dec. 1	Dec. 3
Inflow	400	250	400	150	100	50	50	50	150	400	400	50	0
Outflow	200	200	200	200	200	400	100	100	200	200	200	300	0
Netflow	+200	+50	+200	-50	-100	-350	-50	-50	-50	+200	+200	-250	0

All revenues are received by the district on the first day of the month.

All expenditures occur on the first day of the month.

The beginning cash balance of the district on the first day of the year is $50,000.

An investment process similar to this one is typically followed by most school districts. However, if the school district determined its financial management plan in this manner, it would lose substantial additional revenues through a combination of lost interest and overpayments on borrowed funds. The crucial question for the financial planners to answer is: What investments and borrowings should be made in order to maximize net interest earnings while making sure that all bills are paid? To answer this question, integer programming may be used.

A Model of School District Financial Management

In the previous section, the cash flow pattern and financial management processes typically used by school district financial planners were discussed. In this section, a general model of school district financial management, along with the assumptions of the model, and limitations effecting financial management incorporated in the model will be presented.

Using the cash flow pattern of the example school district given in Table 1, it is evident that there are ample opportunities for the investment of surplus funds, as well as the need to borrow in times of cash deficit. The question is, what investments and borrowings should be made in order for the school district to optimize the effectiveness of the school districts' financial management program. A measure of effectiveness is needed. The purpose of any finanical management program is to make as much net interest revenue for the district as possible while making sure all expenses are met, The logical measure of effectiveness should be stated in terms of dollars. Thus the purpose, or objective function, of this model of financial management is to maximize the net interest for the school district. This objective function may be stated mathematically as:

$$\text{MAX } Z = \sum_{t=0}^{11} \sum_{k=1}^{12-t} (ID_{tk} - IP_{tk})$$

where: Z is the net interest earned through financial management,

ID_{tk} is the interest due on investments made in month t and k is the number of months that will pass until the investment matures,

and IP_{tk} is the interest payable on money borrowed in month t and k is the month in which the cash borrowed must be repaid.

For example, if January is month 0, and April is month 3, an investment of $50,000 made in January and maturing in April (as suggested in the previous section) will earn interest equal to ID_{03}. Thus, in this example, there are 177,147 different investment opportunities. In a like manner, IP_{29} represents the interest payable on a sum of cash borrowed in month 2 which must be repaid 9 months later. Thus, the maximum net interest Z is to be found by summing the interest on all possible investments for all periods minus the interest on all borrowed cash for all periods. These interest earnings and payments will not be known until the integer programming solver determines the optimal investments and borrowings that need to be made. Thus, one of the purposes of this model is to determine what investments and borrowings need to be made.

In the process of making investments, and/or borrowing cash, there are constraints within which the financial managers have to operate. Thus, this objective function of maximizing net interest is subject to the following constraints.

Constraint 1 - The Cash Balance Constraint

In any given month there are a number of financial transactions that occur. There is typically a cash balance left from the previous period which may be utilized in the present period. In period t, this cash balance is denoted as C_{t-1}. The cash balance in any given month, then, is denoted as C_t. Thus, the cash balance left at the beginning of June is denoted as C_5. The beginning cash balance for the year of $50,000 is considered as an inflow in period 0.

Each month there are also cash inflows and cash outflows. These are denoted as I_t and O_t respectively. Thus, the cash inflow for August = I_7 = \$50,000, and the cash outflow for November = O_{10} = \$200,000 (See Table 1 for these figures). These beginning cash balance, inflow, and outflow figures are not generated by the model, and are given by the business staff of the school district.

There are also a number of financial considerations besides the inflows and outflows, and cash balances. In any given month the school district may make investments. Each investment may be denoted by NP_{tk} representing "notes purchased". The subscripts t and k represent the same notation as used in ID_{tk} and IP_{tk} previously discussed. Besides investing cash, the school also may need to borrow cash. The amount of cash borrowed each month is denoted as B_{tk} and the interest due at maturity denoted as IP_{tk}. Investments mature which results in cash returning to the school district. A maturing investment is denoted as $NP_{t-r,r}$ where r is the number of periods that have past since the investment was made. The interest due on investments is likewise denoted as $ID_{t-r,r}$. Finally, borrowed money may be repaid on any month. Borrowed cash which is repaid is denoted as $B_{t-r,r}$ and the interest on borrowed cash is $IP_{t-r,r}$.

This cash balance constraint may be formulated mathematically as:

$$\sum_{k=1}^{12-t} (NP_{tk} - B_{tk}) + \sum_{r=1}^{11} (B_{t-r,r} + IP_{t-r,r}) - \sum_{r=1}^{11} (NP_{t-r,r} + ID_{t-r,r}) +$$

$$C_t - C_{t-1} = I_t - O_t.$$

This mathematical expression may be explained as follows. For any period t, the sum of all cash used for investments or used to pay back loans for all periods in the future, plus the sum of the principal and interest paid back on borrowed funds, minus the sum of the principal and interest on investments maturing that period, plus the ending cash balance for that period, minus the cash balance from the

previous period, must be equal to the inflows minus the outflows for that period.

Constraint 2 - The "Arbitrage" Constraint

Because money that is loaned to tax-exempt institutions is not taxed, the interest rate paid by school districts on money they borrow for short terms is low compared to the interest rates they could obtain by investing money. Consequently, a school district could borrow money for a low interest rate, and invest that money in investments that yield a higher rate, thus making a "profit". This practice is known as arbitrage. Because this practice results in reduced tax revenues for the Federal Government, it is often prohibited or restricted by statuatory law in many states.

This restriction is taken into account in this integer programming model by the inclusion of a constraint which limits the amount of cash that may be borrowed to the amount of the deficit for that period. This constraint is expressed mathematically as:

$$\sum_{k=1}^{12-t} B_{tk} \leq \max (0.0, O_t - I_t)$$

This constraint states that for any period, the sum of all borrowed cash must be less than or equal to 0.0, or the difference between the outflow and inflow for that period, whichever is larger. Thus, if the inflow is larger than the outflow, $O_t - I_t$ will be negative (less than zero). The result will be that the total amount of borrowed cash will be zero dollars. However, if the outflow is greater than the inflow, $O_t - I_t$ will be positive. This allows the model to determine a set of loans which will equal the amount of the deficit. This constraint is entered into the model as an integer (0 or 1) variable. If the constraint is entered as 1, the constraint is in force, and the school district will not take advantage of arbitrage opportunities.

Constraint 3 - Limit on Borrowing with Outstanding Investments

Because of the arbitrage opportunity mentioned above, the school district may attempt to borrow cash every time the inflows for the period exceed the outflows resulting in a deficit situation. However, it is possible that the school district may have investments in its portfolio which have not as yet matured. Constraint 3 stipulates that if the school district has investments outstanding, it may not borrow cash until all investments have been cashed in. This constraint is expressed as two mathematical equations:

$$\sum_{j=1}^{t} \left(\sum_{k=t-j+1}^{12-j} NP_{jk} \right) - D_t \cdot M \leq 0.0 \qquad a.$$

and

$$\sum_{j=1}^{t} \sum_{k=t-j+1}^{12-j} B_{jk} + D_t \cdot M \leq M \qquad b.$$

where: t is the present period,

j is some period that precedes period t,

NP_{jk} is an investment purchased in period j which matures in period t,

D_t is an endogenously determined dichotomous variable (0 or 1) whose value denotes that there are outstanding investments in period t, ($D_t = 1$) or that there are none, ($D_t = 0$),

M is an arbitrary constant with a very large value (i.e., set equal to the total expenditure of the district),

and B_{jk} represents the set of loans which may be taken.

Operationally, these expressions may be interpreted as follows:

a. for any period, if there are investments made in a previous period which matures in a future period, a "flag" is created by the model in the form of the 0 or 1 variable, and

b. if the "flag" for that period indicates that there are outstanding investments, there must be no borrowing for that period until all investments are cashed in.

Again, this constraint may or may not be included in the model depending upon its appropriateness by means of an integer variable (0 or 1), (i.e., if this constraint is entered as 1, this constraint is in effect).

Constraint 4 - The Term Structure of Interest Rates

In the financial markets it is known that the longer money is invested, the higher interest rates will be. For example, an investment of one month's duration may earn only 7 percent annual interest, while a two month investment may earn 7¼ percent annual interest. Therefore, the term structure of the interest rates must be specified for the periods under analysis. The term structures for both investments and borrowing for the periods must be specified. Mathematically, these terms structures are expressed as:

for investments - $$\sum_{k=1}^{12-t} (ID_{tk} - a_k NP_{tk}) \leq 0.0$$

for borrowing - $$\sum_{k=1}^{12-t} (-IP_{tk} + b_k B_{tk}) \leq 0.0$$

where: a_k is the investment interest rate for investments that mature in k months,

b_k is the borrowing interest rate for loans that are due in k months.

For the sample problem, the interest rates for borrowing and investing for varying maturities is given in Table 2. For example, if an investment of $100,000 was made in period 1 and matured in period 7, this investment, NP17 would mature six months after it began. From Table 2 it is stated that an investment of six months' duration would earn 8.25% annually compounded annually. For an investment of k=6

periods duration, the total interest earned would be equal to .04125NP17 or $100,000 (.04125) = $4,125. For this simplified example, the interest rate on investments increases ½% for each period, while the borrowing rate stays constant at 6%.

Constraint 5 - Borrowing Period Constraint

In some states, school districts are constrained by statutory law to borrow only a specified number of times a year. Some states allow school districts to borrow cash as often as they need it, while other states allow borrowing only once in the fiscal year. This constraint is expressed:

$$\sum_{k=1}^{12-t} B_{tk} - F_t(M) \leq 0.0 \qquad \text{a.}$$

$$\sum_{t=1}^{11} F_t \leq N \qquad \text{b.}$$

where F_t is an endogenously determined variable (0 or 1),

M is an arbitrary constant set to a larger value,

and N is an exogenous variable denoting the number of times that borrowing may occur during the fiscal year.

If borrowing is needed in period t, a "flag" F_t is determined by the model through equation a. If $F_t = 1$ denoting that borrowing is required for that period, then the model begins summing F_t. Expression b indicates that the sum of F_t for all periods must be less than or equal to N, the number of times that the school district is eligible to borrow.

In summary, all these constraints with the addition to the objective function constitutes the integer programming model to optimize school district financial management. These constraints and objective function are summarized in Table 3. Besides listing the term structure of interest rates on borrowing and investing, Table 2 also states the inflows (I_t) and outflows (O_t) for each period, as well as the beginning cash balance for the initial period.

TABLE 2.

The Term Structure of Interest Rates and Cash Flows of the Sample Dsitrict

Duration in periods (k)	Borrowing Annual Rate %	Borrowing k Period Rate %	Investment Annual Rate %	Investment k Period Rate %
1	6.00	.005	7.00	.00583
2	6.00	.01	7.25	.01208
3	6.00	.015	7.50	.01875
4	6.00	.02	7.75	.02583
5	6.00	.025	8.00	.03333
6	6.00	.03	8.25	.04125
7	6.00	.035	8.50	.04958
8	6.00	.04	8.75	.05833
9	6.00	.045	9.00	.0675
10	6.00	.05	9.25	.07708
11	6.00	.055	9.50	.08708
12	6.00	.06	9.75	.0975

Period	Cash Inflows (I_t)	Cash Outflows (O_t)	Beginning Cash Balance (C_t)
0	400,000	200,000	50,000
1	250,000	200,000	
2	400,000	200,000	
3	150,000	200,000	
4	100,000	200,000	
5	50,000	400,000	
6	50,000	100,000	
7	50,000	100,000	
8	150,000	200,000	
9	400,000	200,000	
10	400,000	200,000	
11	50,000	300,000	
12	0	0	

TABLE 3.

An Integer Programming Model for the Optimization of School District Financial Management

The Objective Function:

$$\text{MAXIMIZE} \quad Z = \sum_{t=1}^{11} \sum_{k=1}^{12-t} (ID_{tk} - IP_{tk})$$

Subject to the Following Constraints:

1. $$\sum_{k=1}^{12-t} (NP_{tk} - B_{tk}) + \sum_{r=1}^{11} (B_{t-r,r} + IP_{t-r,r}) - \sum_{r=1}^{11} (NP_{t-r,r} + ID_{t-r,r})$$

$$+ C_t - C_{t-1} = I_t - O_t.$$

2. $$\sum_{k=1}^{12-t} B_{tk} \leq \text{MAX}\ (0.0,\ O_t - I_t).$$

3. $$\sum_{j=1}^{t} \left(\sum_{k=t-j+1}^{12-j} NP_{jk} \right) - D(M) \leq 0.0$$

$$\sum_{j=1}^{t} \sum_{k=t-j+1}^{12-j} B_{jk} + D(M) \leq M.$$

4. $$\sum_{k=1}^{12-t} (ID_{tk} - a_k NP_{tk}) \leq 0.0$$

$$\sum_{k=1}^{12-t} (-IP_{tk} + b_k B_{tk}) \leq 0.0.$$

5. $$\sum_{k=1}^{12-t} B_{tk} - F_t(M) \leq 0.0$$

$$\sum_{t=1}^{11} F_t \leq N.$$

Empirical Analysis and Results of the Model

The integer programming formulation of the financial management decision previously described was empirically tested using the data listed in Table 1. An Integer Programming code called MIPZ1 which is a 0-1 mixed integer programming code available at the Purdue University Computing Center was used to test the model. Other programming codes sufficient to solve this problem are generally available at computer service centers and universities.

In order to operationalize the model, the monthly inflows, outflows, the term structure of interest rates, and the beginning cash balance for the sample school district were required to enter into the model. This data for the sample problem is listed in Table 2. For implementation of this model in an actual school district setting, this data could be acquired during the budgeting period while planning for the next fiscal year. As stated previously, the revenues and expenditures may be anticipated with a high degree of certainty. The term structure of interest rates may be obtained through the local banker.

In addition to the data requirements listed above, the financial environmental constraints faced by the school district must be determined. These data requirements include information on the number of periods under analysis, the number of times the school district can borrow cash each fiscal year, whether the district can borrow more than is required during any period, and whether the school district can borrow and invest cash at the same time. Thus, in addition to the data listed in Table 2, the following parameters were entered as data to the model:

1. there are 12 periods in the planning year, each period being one month,
2. the school district is allowed to borrow cash up to 11 times per year,
3. the school district may not borrow with outstanding investments,
4. the school district may not borrow more than the extent of the cash deficit for each period.

Using the data and constraints listed above, the integer programming formulation was tested empirically, the results being listed in Table 4. Table 4. lists the required loans and investments as determined by the model that would optimize the net interest earnings of the sample school district. As shown in the table, it was necessary for the district to borrow three times for a total of $137,834. Recalling the notation used for borrowing and investing, B63 denotes that a loan was required on July 1 for $37,834.80. The loan would be repaid three months later with $567.52 interest due. The total interest on the three loans was $1,317.52. The reason that B63 is such an unusual amount is because the model used the interest earned from investments made in months 0 through 5 to reduce the amount of the deficit of $50,000 for month 6 as shown on Table 1.

The model also resulted in making nine investments of varying duration. The total investments totaled $760,847.70, with a resultant interest earning of $14,150.28. The net interest earning of the model was $12,832.76. This represents the optimal interest earning possible while ensuring that all expenditures are met, and the financial decision making was within the constraints of the model listed previously.

The computer time required for the solution of this problem was 10.58 CPU seconds on a CDC 6500 at the Purdue University Computing Center. The cost of the run was $3.84. The model was also tested varying the constraints. When the "arbitrage" constraint is lifted (ie. the school district may borrow and invest at the same time) the integer programming model degenerates into a linear programming problem. This linear programming problem required substantially less CPU time (1.88 seconds vs. 10.58 seconds for the integer programming problem), and the cost was also reduced to $.92. Changing the number of times per fiscal year borrowing is allowed did not

TABLE 4.

The Results of the Integer Programming Formulation

Borrowing:

Borrowing period and duration	Borrowing Amount	Borrowing Interest	Borrowing Interest Amount
B63	$37,834.80	IP63	$567.52
B72	$50,000.00	IP72	$500.00
B81	$50.000.00	IP81	$250.00
Total	$137,834.80	Total	$1,317.52

Investing:

Investing Period and Duration	Investment Amount	Investment Interest	Investment Interest Amount
NP05	$238,316.70	ID05	$7,943.10
NP06	11,683.30	ID06	481.94
NP14	50,000.00	ID14	1,291.50
NP21	49,710.20	ID21	289.81
NP22	98,806.40	ID22	1,193.58
NP23	51,483.40	ID23	965.31
NP92	48,251.10	ID92	582.87
NP93	12,596.60	ID93	236.17
NP10,1	200,000.00	ID10,1	1,166.00
Total	$760,847.70	Total	$14,150.28

affect the computer time or costs substantially. However, as the number of periods increased, as would happen if the model were a bi-weekly or weekly model instead of monthly, the computer time and costs increased almost geometrically. For an integer programming problem with the "arbitrage" constraint in force, the CPU time for a 12 period model was 10.58, for a 24 period model, the CPU requirement was 26.81 seconds, and for a 52 period, weekly model, the CPU seconds required was 77.3. This is because as the number of periods increase, the number of equations for each class of constraints also increases. That is, if there are 52 periods, there has to be 52 equations entered into the model for each class of constraints. Thus, for the 52 period model, there are 260 equations required, and the number of elements in the objective function is 2,756! The computer cost for the 52 period model was $28.06. A further cost consideration is that the programming costs of developing the matrix generator for the integer program, in addition to the author's time, consisted of approximately 10 hours of a computer programmer's time. However, once the program is written initially, it may be used over again with different data. Thus, the more the program is used, the lower the cost per program. This also permits flexibility in testing the assumptions of the model to obtain the investment and borrowing package which best fits the financial environment faced by the school district.

These costs may be substantially reduced if the financial planners are only concerned with the investment side of the optimization program. Without borrowing, the program degenerates into a linear program with significant reductions in computer time and costs, data requirements, and in the degree of sophistication required to operate the program. For an example of the linear program which is interested only in the investment

aspects of financial management, see Dembowski, 1979.

It may be argued that heuristic methods will produce results nearly as good as those produced by this model. In order to test this argument, two procedures were used. First, the cash flow pattern of the sample school district listed in Table 1. was given to several practicing business managers in local school districts. These managers were also given the knowledge of the constraints used, and the term structure of interest rates incorporated in the model. Their task was to develop a schedule of investments and loans similar those produced by the model listed in Table 4. In all cases, the model produced substatially better results than the manual methods used by the business managers. For example, one test case produced a net interest earning of $11,907 versus the net interest earning of the model of $12,833. Next, these business managers were asked to provide the required data from their districts for use as test cases of the model. The results of the model were then compared with the actual results of the cash management program of the school district for that year. In all cases, the results of the model were at least 10% higher than the actual results of their financial programs, one case having a difference of over $100,000!

Summary of the Integer Programming Model

The integer programming model was developed with the intent to be used as a tool in the decision making process of school district financial planning. As shown by the results of the model in Table 4., the model produces several types of information:

1. the number of investments, with resultant interest that should be made,
2. the number of times borrowing is required, with resultant interest due,
3. the timing, duration, and amounts of borrowing and investments, and

4. the financial package that optimizes the net interest earnings of the school district.

The model is general in nature. The constraints used may be adjusted for inclusion to the model to most closely adheres to the financial environment of the school district. The constraints include arbitrage opportunities, the number of times the school district may borrow cash, and the amount of cash that may be borrowed. The inflows and outflows entered into the model may be adjusted to result in a conservative financial package which understates the amount of cash available for investment, or it may be very "tight", resulting in the investment of all surplus cash.

The model may also be used as a research tool to investigate the cost of the various constraints and limitations placed on the school district by the environment. For example, the model may be used to examine how much interest revenue is lost to the school district because of the arbitrage constraint. This could be accomplished by running the model once with the constraint, and once without, tehn evaluating the net interest revenue difference. Or, the effect of a conservative vs. a liberal investment policy may be examined by adjusting the inflows and outflows entered into the model.

With budgets of several million dollars, school districts have an opportunity to develop substantial revenues through wise financial management. Through the use of this model, as well as other financial techniques available, school district financial managers may better utilize this financial resource. Increased efficiency in this management area will not only help aid to ease the financial crisis faced by many school districts, but will help the districts meet the expectations of their taxpaying constituents.

REFERENCES

1. Calman, R.F., Linear Programming and Cash Management: CASH ALPHA, Cambridge, Mass., The MIT Press, 1968.

2. Dembowski, F.L., "A Model For Determining Cash Flow Needs'', a presentation at the Annual Conference of the Association of School Business Officials of the U.S. and Canada, Denver, Colorado, Oct., 1979.

3. Dembowski, F.L., "The Optimization of School District Cash Management: A Case Study", an unpublished doctoral dissertation, The University of Rochester, 1978.

4. Dembowski, F.L., "Use Your Cash Management Potential", Educational Economics, July/August, 1978.

5. Elton, E. and Gruber, M., "Dynamic Programming Applications in Finance", Journal of Finance, May 1971.

6. Fox, K.A. and Sengupta, J.K., "The Specification of Economative Models for Planning Educational Systems", Kyklos, vol. 21, 1968.

7. Hausman, W.H., and Sanchez, Bill, A., "The Stochostic Cash Balance Requirements", Management Science, vol. 21(8), 1975.

8. McNamara, J.F., "Mathematical Programming Models in Educational Planning", Review of Educational Research, 41(5), 1973.

9. Merville, L.J., and Tavis, L.A., "Optimal Working Capital Policies: A Chance Constrained Programming Approach", Journal of Financial and Quantitative Analysis, Jan., 1973.

10. Orgler, Y.E., "An Unequal Period Model for Cash Management Decisions", Management Science, 16(2), 1969.

11. Pogue, G.A., and Bussard, R.N., "A Linear Programming Model for Short Term Financial Planning Under Uncertainty", Sloan Management Review, Spring, 1972.

12. Robichek, A.A., Teichroew, D., and Jones, J.M., "Optimal Short-Term Financing Decision", Management Science, Sept., 1965.

13. Searby, F.W., "Use Your Hidden Cash Resources", Harvard Business Review, March/April, 1968.